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Division of Electromagnetic Research

Geometric Optics and the Polarization Vectors

Robert M. Lewis

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Research Report No. EMp-209

Contract No. AF 19(628)4065
Project No. 4603
Task No. 460302

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C. 1

Abstract

Classical geometrical optics provides simple rules for the construction of electromagnetic fields. However the rules are incomplete in that the directions of E and H are not completely determined. The usual rules can be derived from an asymptotic solution of Maxwell's equations. This process also yields equations for the direction of E and H. These equations are shown to yield a very simple direction rule which completes the geometrical optics determination of the fields.

Geometrical Optics and the Polarization Vectors

Robert M. Lewis

Classical geometrical optics provides simple rules for the construction of the rays (Fermat's principle), phase (optical distance along a ray), and scalar amplitude (energy conservation in a tube of rays) that determine a geometrical optics field. However the rules are incomplete in that the directions of the electric and magnetic field vectors are not prescribed. It is well known that the geometrical optics field is the first term in a high-frequency asymptotic expansion of the electromagnetic field, and the above construction rules can be derived from Maxwell's equations. This derivation also yields the following prescription for the direction of the electric and magnetic fields of linearly polarized waves in isotropic inhomogeneous media. [1] Let T, N, and B be unit tangent, normal, and binormal vectors of a ray. Let P and Q be unit vectors in the direction of the electric and magnetic fields. Then

$$P = (\cos\alpha)N + (\sin\alpha)B, \quad Q = -(\sin\alpha)N + (\cos\alpha)B \tag{1}$$

and

$$\frac{d\alpha}{ds} = -\tau \ . \tag{2}$$

Here s denotes arc length along a ray and τ is the torsion of the ray. These equations imply that T, P, and Q form a right-handed orthonormal set, which we may call the polarization trihedral, which rotates relative to the Frenet trihedral, T, N, and B in a manner determined by (2).

That T, P, and Q form a right-handed orthonormal set is a simple rule that we could append to the above construction rules. But the rotation law in its customary formulation (2) obscures the fact that there is a very simple rotation rule, equivalent to (2) that can be added to the construction rules in order to complete the geometrical optics prescription of the field.

The rule is simply that the polarization trihedral does not rotate about the ray. In more precise terms, let us identify s with the time t and introduce the angular velocity vector Ω_p of the polarization trihedral. Then we will prove that

$$\Omega_p \cdot T = 0 , \tag{3}$$

i.e. Since Ω_p has the direction of the instantaneous axis of rotation, there is no component of rotation about the ray (in the direction of T.)

In order to prove (3) we denote the curvature of the ray by κ and introduce the Frenet equations

$$\dot{T} = \kappa N , \quad \dot{N} = -\kappa T + \tau B , \quad \dot{B} = -\tau N . \tag{4}$$

Then we differentiate (1) with respect to t = s by using (2) and (4). The result is

$$\dot{P} = -\tau Q + \cos\alpha\left[-\kappa T + \tau B\right] + \sin\alpha\left[-\tau N\right] = -\kappa(\cos\alpha)T, \tag{5}$$

$$\dot{Q} = \tau P - \sin\alpha\left[-\kappa T + \tau B\right] + \cos\alpha\left[-\tau N\right] = \kappa(\sin\alpha)T. \tag{6}$$

Thus T, P, and Q satisfy the system of equations

$$\dot{T} = (\kappa \cos \alpha)P - (\kappa \sin \alpha)Q, \quad \dot{P} = -(\kappa \cos \alpha)T, \quad \dot{Q} = (\kappa \sin \alpha)T. \tag{7}$$

Now if $\Omega = \omega_1 U_1 + \omega_2 U_2 + \omega_3 U_3$ is the angular velocity of the moving trihedral U_1, U_2, U_3, then from the well known properties of Ω

$$\left.\begin{aligned} \dot{U}_1 &= \Omega \times U_1 = 0 + \omega_3 U_2 - \omega_2 U_3 \\ \dot{U}_2 &= \Omega \times U_2 = -\omega_3 U_1 + 0 + \omega_1 U_3 \\ \dot{U}_3 &= \Omega \times U_3 = \omega_2 U_1 - \omega_1 U_2 + 0 \end{aligned}\right\} \tag{8}$$

If we now compare (7) and (8) we find that $\omega_1 = 0$, $\omega_2 = \kappa \sin \alpha$, and $\omega_3 = \kappa \cos \alpha$, i.e. the angular velocity Ω_p of the polarization trihedral is given by

$$\Omega_p = \omega_1 T + \omega_2 P + \omega_3 Q = (\kappa \sin \alpha)P + (\kappa \cos \alpha)Q = \kappa B \tag{9}$$

which implies (3). Furthermore it is not difficult to show that the argument leading to (3) can be reversed, i.e. that (3) implies (1) and (2). It is interesting to observe that the angular velocity Ω_f of the Frenet trihedral, which can be obtained by comparing (4) and (8) is given by

$$\Omega_f = \tau T + \kappa B. \tag{10}$$

Thus if $\tau \neq 0$, the Frenet trihedral has a component of rotation about the ray.

It is amusing to recall that the polarization trihedral is ordinarily described as rotating about the ray relative to the Frenet trihedral at a rate determined by (2). Although this is true we see that in fact the Frenet trihedral rotates in the opposite direction at such a rate that the rate of rotation of the polarization trihedral about the ray is zero.

[1] Kline, Morris and Kay, Irvin W.: Electromagnetic theory and Geometrical Optics, John Wiley & Sons, New York, 1965, pp. 167-170.

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